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Theoretical analysis of a centre-fed non-planar dipole antenna in weakly ionised plasma

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The presence of plasma has a profound effect on the properties of antenna. A generalised analysis of the non-planar dipole antenna in plasma medium is presented. Mathematical expressions for the far field radiation patterns are developed and calculations are made to predict the effect of the centre-fed half-wave and full-wave dipole antenna immersed in a weakly ionised plasma. Typical plots and tables are given to elucidate the theoretical results.

1. Introduction

In recent communications (Kosta 1969, 1967) theoretical studies are made to investigate the effect of feed point displacement on the centre-fed dipole antenna and important applications of feed displacements have been pointed out. Kosta (1970) has also treated in detail the effect of feed points displacement on the radiation patterns of a centre fed dipole antenna, immersed in a weakly ionised plasma medium. Using the concept of inclined linear antenna, a non-planar dipole antenna (NPD) has been investigated (Singh 1971). The specific features of this antenna configuration is that it acts as a source of complex radiation, i.e., the field radiated by the antenna gives three components of E vector along the three orthogonal axes, while a simple linear dipole gives only one E vector either horizontal or vertical, depending upon the orientation of the dipole. From the theoretical analysis and experiments, it is found that the E-plane pattern for the NPDA is not as sharp a that of the conventional dipole pattern for any value of displacement. The H plane pattern for the conventional dipole is omnidirectional but that of the NPDA with transversely and arbitrarily displaced feed points, it is elliptical for any value of displacement. In this paper it is proposed to study theoretically the radiation pattern of a centre fed non-planar dipole antenna immersed in weakly ionised compressible warm plasma. In a warm compressible plasma there are, in general, three types of waves on the surface of the antenna : electromagnetic (EM), electro-acoustic and surface waves (Wunsch 1968). Out of these, surface wave is non-radiating and exists at all frequencies if the plasma is compressible. Further, in a warm continuum fluid plasma the electromagnetic and electro-acoustic fields remain uncoupled. Therefore, for analysis, these fields can be separated into two independent modes, viz., *EM* and *P* modes respectively. A linearized theory is used so that the isotropic electron plasma may be regarded as a single fluid continuum. The effects of the characteristics of the non-planar dipole antenna have been discussed for three values of propagation constant, $\beta = \beta_0$, $\beta = \beta_e$ and $\beta = \beta_p$.

2. Mathematical Analysis

The presence of the plasma affects not only the form of the current distribution on the antenna, but it also alters the propagation constant for the current distribution. Recent theoretical studies by Lin & Mei (1968) and by Wunsch (1968) suggest that the current distribution on a thin resonant antenna immersed in an isotropic plasma can be assumed to be sinusoidal for $(\omega_p/\omega)^2 \geqslant 0.80$. But there are different opinions about the value of the propagation constant. The propagation constant for this current distribution can be taken to be equal to either that of the plasma wave, as is done by Cook & Edgar (1966), or that of the electromagnetic wave, as is suggested by Chen *et al* (1967), and by Judson *et al* (1968), who from their theoretical and experimental studies of linear antennas in a plasma, maintain that a reasonable theoretical models for the antenna current can be obtained by putting $\beta = \beta_e$ in the sinusoidal current distribution for $\omega_p < \omega$. Therefore, it is better to study the radiation properties of antenna in plasma by assuming a general propagation constant, as is done in this paper. The plasma is assumed to be homogenous, neutral and in unperturbed state. The perturbation of the plasma due to the radiating source is also small so that the linearized equations are valid. The plasma is also assumed to be lossless and free of static electric and magnetic fields. The basic equations in spherical polar coordinate for obtaining the electromagnetic and plasma for zone fields of a thin antenna in a warm plasma have been derived by Wait (1964*a*, 1965*a*).

Fig. 1 shows the geometry and the coordinate system of the non planar dipole antenna. The arms AB and CD are inclined by an angle α_1 and α_2 with respect to the Z axis and lies in the X–Z plane and Y–Z plane respectively. On resolving, we get four components. Components (1) and (2) due to the inclined arm AB, while the components (3) and (4) due to the inclined arm CD. The components (1) and (4), being in same axis, can be added to give the total value of the component along the Z axis. Let AB and CD, the two half portions of the antenna which may be considered to consist of a large number of very small elements dz', each have a constant current $I(Z)$. The far field of an element dz' in a lossless plasma is given by

$$dE_e = \frac{j\beta_e Z\, I(Z)\, dz' \sin\theta}{4\pi r_0} \exp(j\omega t - \beta_e r_0) \exp(j\beta_e Z' \cos\theta)$$

where Z is the intrinsic impedance of the plasma medium, Z_0 intrinsic impedance of free space and $\eta = \left(\frac{\mu}{\epsilon_0}\right)^{\frac{1}{2}} = 120\pi$.

To obtain these fields a current distribution is to be assumed which is found to be valid under above mentioned conditions i.e.,

$$I(Z) = I_m \sin \beta(L-Z), \qquad \text{for } Z > 0$$
$$= I_m \sin \beta(L+Z), \qquad \text{for } Z < 0. \qquad \dots \quad (1)$$

where I_m is the current amplitude and $\beta = \frac{2\pi}{\lambda}$ is the general propagation constant.

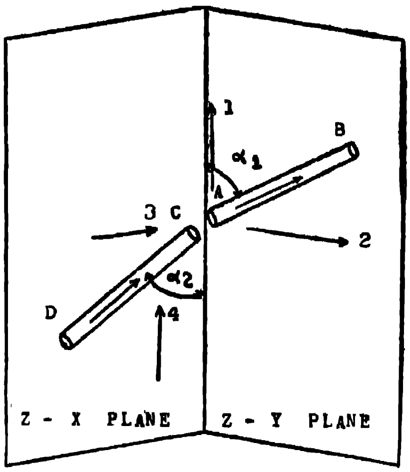

Fig. 1. Layout and coordinate system of the non-planar dipole antenna.

Thus the total far field at a far point is given as

$$|E(\theta, \phi)| = [|E_z(\theta, \phi)|^2 + |E_y(\theta, \phi)|^2 + |E_x(\theta, \phi)|^2]^{\frac{1}{2}}. \qquad \dots \quad (2)$$

where

$$E_z(\theta, \phi) = B \cos \alpha_1 \int_0^L \exp(j\beta_e Z' \cos \theta).\sin \beta(L-Z')dz'$$

$$+B \cos \alpha_2 \int_{-L}^0 \exp(-j\beta_e Z' \cos \theta).\sin \beta(L+Z')dz'. \qquad \dots \quad (3)$$

$$E_y(\theta, \phi) = B' \sin \alpha_1 \int_0^L \exp(j\beta_e Y \sin \theta \sin \phi).\sin \beta(L-Y)dy. \qquad \dots \quad (4)$$

$$E_x(\theta, \phi) = B'' \sin \alpha_2 \int_0^L \exp(j\beta_e X \sin \theta \sin \phi).\sin \beta(L-X)dx. \qquad \dots \quad (5)$$

where

$$B = \frac{j\beta_e Z\, I_m \sin\theta}{4\pi r_0} \exp(j\omega t - \beta_e r_0)$$

$$B' = \frac{j\beta_e Z\, I_m (1-\sin^2\theta \sin^2\phi)^{\frac{1}{2}}}{4\pi r_0} \exp(j\omega t - \beta_e r_0)$$

$$B'' = \frac{j\beta_e Z\, I_m (1-\sin^2\theta \cos^2\phi)^{\frac{1}{2}}}{4\pi r_0} \exp(j\omega t - \beta_e r_0).$$

2.1 *Far Zone Field of EM Wave*

Using the standard integral formula and simplifying the eqs. (3), (4) and (5),

$$\int e^{ax} \sin(C+bx) = \frac{e^{ax}}{a^2+b^2}\{a \sin(C+bx) - b\cos(C+bx)\}$$

we get,

$$|E(\theta,\phi)_e|\,z = \frac{B(\cos\alpha_1+\cos\alpha_2)}{\beta^2-\beta_e^2\cos^2\theta}\{\cos(P)-\cos\beta L\}+j\,\frac{B(\cos\alpha_1-\cos\alpha_2)}{\beta^2-\beta_e^2\cos^2\theta}$$
$$\{\sin(P)-a\sin\beta L\cos\theta\}. \qquad (6)$$

$$|E(\theta,\phi)_e|\,Y = \frac{B'\sin\alpha_1}{\beta^2-\beta_e^2\sin^2\theta\sin^2\phi}[\{\cos(S)-\cos\beta L\}$$
$$+j\{\sin(S)-a\sin\theta\sin\phi\sin\beta L\}]. \qquad (7)$$

$$|E(\theta,\phi)_e|\,x = \frac{B''\sin\alpha_2}{\beta^2-\beta_e^2\sin^2\theta\sin^2\phi}[\{\cos(T)-\cos\beta L\}$$
$$+j\{\sin(T)-a\sin\theta\cos\phi\sin\beta L\}]. \qquad (8)$$

where

$$P = \beta_e(L\cos\theta)$$
$$S = \beta_e(L\sin\theta\sin\phi)$$
$$T = \beta_e(L\sin\theta\cos\phi)$$

Eqs. (6), (7) and (8) give the far zone fields of the *EM* waves of the *NPD* antenna.

2.2 *Far Zone Field of P-Wave*

Using the relations

$$\boldsymbol{H} = Z^{-1}\mathbf{E}$$
$$\boldsymbol{V}\,\boldsymbol{J} = -j\omega q$$

where $\boldsymbol{H}$ is magnetic field vector, $\boldsymbol{E}$ is electric field vector and Z is intrinsic impedance for plasma medium. We obtain

$$q = \left(\frac{j\beta}{\omega}\right) I_m \cos\beta(L-Z') \quad \text{for } 0 < Z' < L. \qquad \ldots (9a)$$

$$q = \left(\frac{-j\beta}{\omega}\right) I_m \cos\beta(L+Z') \quad \text{for } -L < Z' < 0. \qquad \ldots (9b)$$

Using basic equations, the following expressions for the perturbation of the plasma density are obtained,

$$n_{1z} = M[\cos\alpha_1 \int_0^L e^{(j\beta p z'\cos\theta)} \cos\beta(L-Z')dz' - \cos\alpha_2 \int_{-L}^0 e^{(j\beta p z'\cos\theta)}.\cos\ \beta(L+Z')dz'], \quad \dots \quad (10)$$

$$n_{1y} = M[\sin\alpha_1 \int_0^L \exp(j\beta_p Y \sin\theta\sin\phi).\cos\beta(L-Y)dy]. \quad \dots \quad (11)$$

$$n_{1x} = M[\sin\alpha_2 \int_0^L \exp(j\beta_p X \sin\theta\cos\phi).\cos\beta(L-X)dx]. \quad \dots \quad (12)$$

Now using the standard integral formula and carrying out the integration and simplifying, we get

$$n_{1z} = \frac{M}{(\beta^2-\beta_p^2\cos^2\theta)}[(\cos\alpha_1+\cos\alpha_2)\{\beta_p\cos\theta\cos(X)-\beta_p\cos\theta\cos\beta L$$

$$-j(\cos\alpha_1-\cos\alpha_2)\{\beta\sin\beta L-\beta_p\cos\theta\sin(X)\}], \quad \dots \quad (13)$$

$$n_{1y} = \frac{M\sin\alpha_1}{\beta^2-\beta_p^2\sin^2\theta\sin^2\phi}[\{\beta\sin\beta L-\beta_p\sin\theta\sin\phi\sin(Z'')\}$$

$$+j\{\beta_p\sin\theta\sin\phi\cos(Z'')-\beta_p\sin\theta\sin\phi\cos\beta L\}]. \quad \dots \quad (14)$$

$$n_{1x} = \frac{M\sin\alpha_2}{\beta^2-\beta_p^2\sin^2\theta\cos^2\phi}[\{\beta\sin\beta L\sin\theta-\beta_p\cos\phi\sin(V)\}$$

$$+j\{\beta_p\cos\phi\cos(V)-\beta_p\cos\phi\cos\beta L\}]. \quad \dots \quad (15)$$

where

$$M = \frac{-j\omega_p^2\beta I_m}{4\pi r_0\omega_e v_0^2}\exp(j\omega t-j\beta_p r_0)$$

$$X = \beta_p(L\cos\theta)$$

$$Z'' = \beta_p(L\sin\theta\sin\phi)$$

$$V = \beta_p(L\sin\theta\cos\phi)$$

Eqs. (13), (14) and (15) give the far zone fields of the **P-waves of the NPD** antennas.

2.3 *Field Patterns of the Antenna for EM Wave*

An expression for the pattern factor for the EM wave mode of a resonant antenna is obtained by using the relations in terms of electric fields

$$F_{\theta e} = \frac{2\pi}{Z_0}.\frac{r_0}{I_m}|E|, \quad Z = \frac{Z_0}{b}, \quad a = \frac{\beta_e}{\beta}$$

Now eqs. (6), (7) and (8) can be rewritten as,

$$|F(\theta,\phi)_e|_Z = \frac{a\sin\theta}{b(1-a^2\cos^2\theta)}[(\cos\alpha_1+\cos\alpha_2)\{\cos(P)-\cos(2\pi m)\}$$

$$+j(\cos\alpha_1-\cos\alpha_2)\{\sin(P)-a\sin(2\pi m)\cos\theta\}]. \quad \ldots \quad (16)$$

$$|F(\theta,\phi)_e|_Y = \frac{a\sin\alpha_1(1-\sin^2\theta\sin^2\phi)^{\frac{1}{2}}}{b(1-a^2\sin^2\theta\sin^2\phi)}[\{\cos(S)-\cos(2\pi m)\}$$

$$+j\{\sin(S)-a\sin\theta\sin\phi\sin(2\pi m)\}], \quad \ldots \quad (17)$$

$$|F(\theta,\phi)_e|_X = \frac{a\sin\alpha_2(1-\sin^2\theta\cos^2\phi)^{\frac{1}{2}}}{b(1-a^2\sin^2\theta\cos^2\phi)}[\{\cos(T)-\cos(2\pi m)\}$$

$$+j\{\sin(T)-a\sin\theta\cos\phi\sin(2\pi m)\}]. \quad \ldots \quad (18)$$

2.4 *Field Patterns of the Antenna for P Waves*

Expression, for the pattern factor for the P-wave mode of a resonant antenna is obtained from the eqs. (13), (14) and (15) by using the relations

$$a = (\beta_e/\beta), \qquad d = (\beta_p/\beta)$$

and

$$b = \sqrt{\delta} = \sqrt{1-\left(\frac{\omega_p^2}{\omega^2}\right)}, \quad m = L/\lambda$$

$$|F(\theta,\phi)_p|_Z = \frac{e\,c\,d\cos\theta}{m\omega b v_0(1-d^2\cos^2\theta)}[(\cos\alpha_1+\cos\alpha_2)\{\cos(X)-\cos(2\pi m)\}$$

$$-j(\cos\alpha_1-\cos\alpha_2)\{(1/d)\sin(2\pi m)\sec\theta-\sin(X)\}]. \quad \ldots \quad (19)$$

$$|F(\theta,\phi)_p|_Y = \frac{ecd\sin\theta\sin\alpha_1}{m\omega b v_0(1-d^2\sin^2\theta\sin^2\phi)}[\{(1/d)\operatorname{cosec}\theta\sin(2\pi m)-\sin\phi\sin(Z)\}$$

$$+j\{\sin\phi\cos(Z)-\sin\phi\cos(2\pi m)\}]. \quad (20)$$

$$|F(\theta,\phi)_p|_X = \frac{ecd\sin\theta\sin\alpha_2}{m\omega b v_0(1-d^2\sin^2\theta\cos^2\phi)}[\{(1/d)\sin(2\pi m)\operatorname{cosec}\theta-\cos\phi\sin(V)\}$$

$$+j\{\cos\phi\cos(V)-\cos\phi\cos(2\pi m)\}]. \quad \ldots \quad (21)$$

Thus the total field pattern factor for the P-wave is given as

$$|F(\theta,\phi)_p| = [\,|F(\theta,\phi)_p|^2{}_Z + |F(\theta,\phi)_p{}^2)_Y + |F(\theta,\phi)_p|^2{}_X]^{\frac{1}{2}}.$$

From above general expression, E and H plane patterns may be derived by putting proper values of θ and ϕ.

It is apparent that $F(\theta,\phi)_e$ and $F(\theta,\phi)_p$ are functions of β. Thus in order to study the field patterns the value of β is to be specified. In the present case the field patterns have been investigated for three different values of β.

Case I : For $\beta = \beta_0$, the parameters reduce to $\beta L = 2\pi(L/\lambda)$,

$$a = \left(\frac{\beta_e}{\beta_0}\right) = b, \quad d = \frac{c}{v_0} b.$$

Case II : For $\beta = \beta_e$, the parameters reduce to $\beta L = 2\pi b(L/\lambda)$,

$$a = 1, \quad d = \frac{c}{v_0}.$$

Case III : For $\beta = \beta_p$, the parameters reduce to $\beta L = 2\pi b(c/v_0),(L/\lambda)$,

$$a = \frac{v_0}{c}, \quad d = 1.$$

As shown by Chen (1967) an antenna excites plasma waves in the range $0.7 \leqslant \omega_{p1}^2 \leqslant 1$ (where ω_{p1} is the average plasma frequency). Keeping this in view and taking $c/v_0 = 10^3$, the mathematical expressions for $F_{\theta e}$, $F_{\theta p}$ and $F_{\varphi e}$, $F_{\varphi p}$ for the three cases have been analyzed.

3. Conclusions

Mathematical expressions for the far field radiation patterns for the EM and plasma fields of half wave and full wave non-planar dipole antennas have been derived. Theoretical results are shown in Figs. 2 to 4. For the purpose of investigations three case of propagation constants, namely $\beta = \beta_0$, $\beta = \beta_e$ and $\beta = \beta_p$ have been considered. Tables 1 and 2 show the results of the investigations. From these tables and plots following conclusions have been drawn :

For $\beta = \beta_0 = \beta_e$ and $L = \lambda/4$, $\lambda/2$ (length of dipole).

1. The E-plane pattern is figure of eight type with a beam maxima at 90° and the H-plane pattern is circular with slight deterioration. As α_1 and α_2 are increased the E-plane pattern becomes approximately circular.
2. For a constant α_2, the magnitude of E and H plane patterns decreases as α_1 increases.
3. If α_1 and α_2 are kept constant then the amplitudes of the far fields $E(\theta)$ and $(E\phi)$ are more in full-wave NPD than half-wave NPD.

For $\beta = \beta_p$, $L = \lambda/4$, $\lambda/2$

1. The E-plane beam is splitted and each beam has a marked maxima at 36° and 144°.
2. The H-plane pattern is also splitted and takes the shape of bowl.
3. For a constant α_2 the magnitude of E and H plane patterns decreases as α_1 increases.

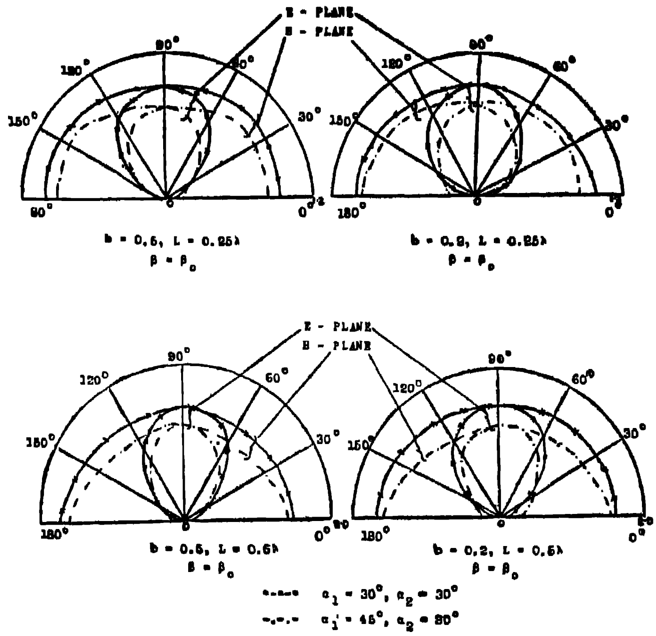

Fig. 2. *E* and *H* plane patterns of non-planar dipole antenna immersed in a weakly ionised plasma for propagation constant $\beta = \beta_0$.

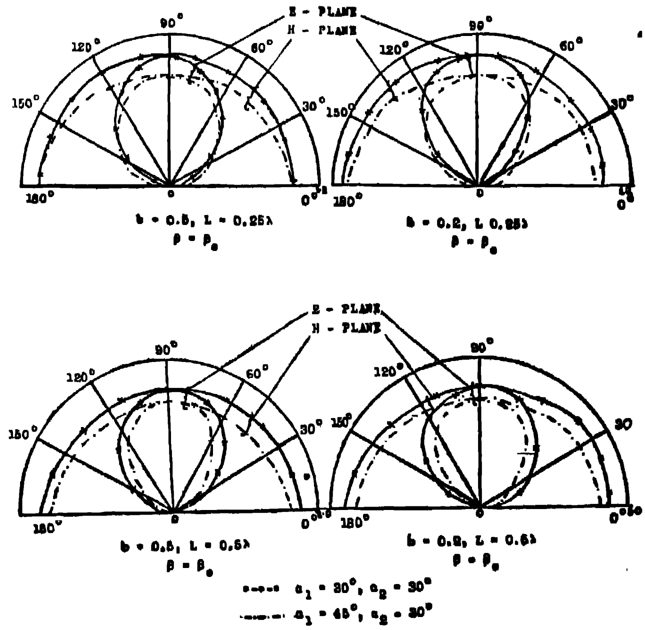

Fig. 3. *E* and *H* plane patterns of non-planar dipole antenna immersed in a weakly ionised plasma for propagation constant $\beta = \beta_e$.

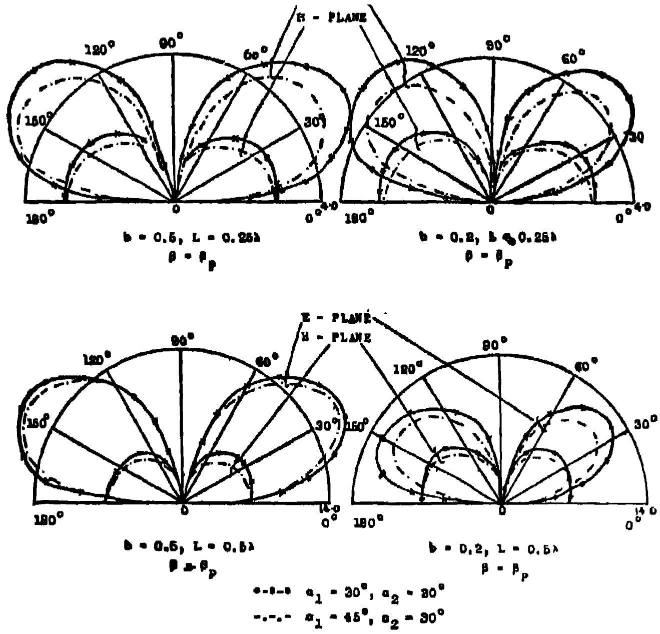

Fig. 4. E and H plane patterns of non-planar dipole antenna immersed in a weakly ionised plasma for propagation constant $\beta = \beta_p$.

Table 1. Study of radiation pattern (F_{0e}) of half-wave non-planar dipole antenna immersed in a weakly ionised plasma

b	Combination of inclination angle		$\beta = \beta_0$		$\beta = \beta_c$		$\beta = \beta_p$		
	α_1(deg)	α_2(deg)	Max. position of the beam (deg)	Beam width (deg)	Max. position of the beam (deg)	Beam width (deg)	Max. position of the right beam (deg)	Max. position of the left beam (deg)	Beam width (deg)
0.2	30	30	90	94	90	90	36	144	30
0.2	30	45	90	100	90	108	36	144	30
0.2	30	60	90	134	90	+	36	144	26
0.2	45	30	90	95	90	108	36	144	36
0.2	45	45	90	134	90	+	36	144	28
0.2	45	60	90	+	90	+	36	144	28
0.2	60	30	90	+	90	+	36	144	27
0.2	60	45	90	+	90	+	36	144	30
0.2	60	60	90	+	90	+	36	144	30
0.5	30	30	90	97	90	89	36	144	34
0.5	30	45	90	108	90	108	36	144	34
0.5	30	60	90	136	90	+	36	144	30
0.5	45	30	90	103	90	110	36	144	32
0.5	45	45	90	138	90	+	36	144	32
0.5	45	60	90	+	90	+	36	144	31
0.5	60	30	90	+	90	+	36	144	30
0.5	60	45	90	+	90	+	36	144	30
0.5	60	60	90	+	90	+	36	144	32

+. The pattern is almost circular.

Table 2. Study of radiation pattern ($F_{\theta e}$) of full-wave non-planar dipole antenna immersed in a weakly ionised plasma

b	Combination of inclination angle		$\beta = \beta_0$		$\beta = \beta_e$		$\beta = \beta_p$		
	α_1(deg)	α_2(deg)	Max. position of the beam (deg)	Beam width (deg)	Max. position of the beam (deg)	Beam width (deg)	Max. position of the right beam (deg)	Max. position of the left beam (deg)	Beam width (deg)
0.2	30	30	90	96	90	98	36	144	34
0.2	30	45	90	108	90	108	36	144	34
0.2	30	60	90	+	90	+	36	144	26
0.2	45	30	90	96	90	108	36	144	30
0.2	45	45	90	+	90	+	36	144	27
0.2	45	60	90	+	90	+	36	144	27
0.2	60	30	90	+	90	+	36	144	30
0.2	60	45	90	+	90	+	36	144	30
0.2	60	60	20	+	90	+	36	144	32
0.5	30	30	90	90	90	90	36	144	34
0.5	30	45	90	92	90	92	36	144	34
0.5	30	60	90	144	90	+	36	144	32
0.5	45	30	90	92	90	92	36	144	33
0.5	45	45	90	108	90	+	36	144	33
0.5	45	60	90	+	90	+	36	144	32
0.5	60	30	90	+	90	+	36	144	34
0.5	60	45	90	+	90	+	36	144	34
0.5	60	60	90	+	90	+	36	144	32

+ The pattern is almost circular.

From the above study it is found that the E plane patterns of the NPDA in free space deviates much from the conventional dipole pattern, in the vicinity of 0° to 30° and 150° to 180° while in plasma media at $\beta = \beta_p$, the E plane pattern of NPDA is splitted. The E plane pattern for the NPDA is not as sharp as that of the conventional dipole pattern. The H plane pattern for the conventional dipole is omnidirectional but that of the NPDA with transversely and arbitrarily displaced feed points is elliptical. In plasma media, the H plane pattern for $\beta = \beta_0 = \beta_e$ is circular with slight deterioration and for $\beta = \beta_p$, the H plane pattern is splitted and takes the shape of the bowl. A further point of interest is that, for a non-planar dipole antenna in free space, the direction of the main lobe can be altered merely by altering the value of displacement, over a range of θ from 85° to 100°, without altering the magnitude of the lobe. This could be used as a means of steering antennas. The problem of high speed transmission/reception of various types of data with a possibility of gradual change in the polarisation direction of the signal in the long space path, demanded an antenna capable of providing a wide band performance with dual polarisation characteristics. A non planar dipole configuration is one of the solutions to this problem. Thus NPD may find important applications in the space communications and defence problems.

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